647510 **BSCmaMN202x-2C** **Seat No :_________**

B.Sc. Semester - 2 (NEP-2020) Examination
MARCH/APRIL-2026 (As Per Syllabus Year June-2023)
MATH: Theory of Ordinary Differential Equations (Minor-2)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any ONE. (05)

(1) Obtain the differential equation for the equation $y = \alpha e^{-2x} + \beta e^{3x}$. What will be the order and degree of this differential equation ?

(2) Solve the differential equation $(x^2 - 2y^2)dx - xydy = 0$.

Que.1(B) Answer any ONE. (05)

(1) State and prove the necessary and sufficient conditions for a differential equation to be exact.

(2) What is an orthogonal trajectory of a family of curves? Obtain it for the family of curves given by $x = ky^3$, $k \in \mathbb{R}$.

Que.2(A) Answer any ONE. (05)

(1) Obtain general solution of $y\left(\frac{dy}{dx}\right)^2 - x(5y+1)\frac{dy}{dx} + 5x^2 = 0$.

(2) Solve the initial value problem $y + px = p^2x^4$, $y(1) = 2$.

Que.2(B) Answer any ONE. (05)

(1) State the general form of Lagrange's differential equation. What is its order? Is it linear ? Discuss it's solution.

(2) Solve : $p^4 - xp - y = 0$.

Que.3(A) Answer any ONE. (05)

(1) Discuss the solution of differential equation $f(D)y = 0$, where $f(D)$ is a polynomial over $\mathbb{R}$ having all distinct real roots.

(2) Solve : $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 4\frac{dy}{dx} = 0$.

Que.3(B) Answer any ONE. (05)

(1) Prove : $\frac{1}{f(D^2)} \cos ax = \frac{1}{f(-a^2)} \cos ax$, if $f(-a^2) \neq 0$.

(2) Prove: If $f(D)y = e^{ax}$ is given linear differential equation with constant coefficients and $f(D) = (D-a)\, g(D)$ with $g(a) \neq 0$, then $\frac{1}{f(D)}\, e^{ax} = \frac{x \cdot e^{ax}}{g(a)}$.

Que.4(A) Answer any ONE. (05)

(1) Find the complementary function for $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 20y = (x+1)^2$.

(2) Find particular integral of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^m$. Discuss all cases arising due to m.

Que.4(B) Answer any ONE. (05)

(1) If operator $\theta = x\frac{d}{dx}$ and $f(\theta)$ is a polynomial over $\mathbb{R}$ with $m \in \mathbb{R}$ such that $f(m) \neq 0$, then prove that $\frac{1}{f(\theta)}\, x^m = \frac{1}{f(m)}\, x^m$.

(2) Find particular integral for $\frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{dy}{dx} = \frac{12\, \log x}{x^2}$.

Que.5(A) Answer any ONE. (05)

(1) Solve : $(x^2 + y^2 - a^2)\, xdx + (x^2 - y^2 - b^2)\, ydy = 0$.

(2) State Clairaut's differential equation and obtain its general solution and singular solution.

Que.5(B) Answer any ONE. (05)

(1) Solve : $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = \sin 2x$.

(2) Find the complementary function for $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$.

****